Journal of Radioanalytical and Nuclear Chemistry, Vol. 261, No. 3 (2004) 637–643

Universal curve of the thermal neutron self-shielding factor in foils, wires, spheres and cylinders

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(Received March 9, 2004)

The interpretation of the sample activation in a nuclear reactor requires the knowledge of two corrective parameters: the thermal neutron self-shielding factor, G_{th}, and the resonance neutron self-shielding factor, G_{res}. The authors established a universal curve of G_{res} for isolated resonances and various geometries. The present paper deals with the description of G_{th} in foils, wires, spheres and cylinders by means of a universal curve on the basis of a dimensionless variable which includes the physical, nuclear and geometrical properties of the sample. The universal curve is in good agreement with the experimental and calculated values obtained from the literature.

Introduction

The irradiation of a sample in the neutron field of a nuclear reactor is affected by the local perturbation of the neutron fluxes produced by the sample.[1] In general, the interpretation of the sample activation due to thermal and epithermal neutrons requires the knowledge of two corrective parameters: the thermal neutron self-shielding factor, G_{th}, and the resonance neutron self-shielding factor, G_{res}.

In previous works, the authors established a universal curve of G_{res} for isolated resonances or groups of isolated resonances, and various geometries of the samples.[2–4]

In relation to G_{th}, some experimental and theoretical studies have been carried out to determine this factor for foils and wires of different elements.[5–14] The corresponding results are presented as tables or graphics of G_{th} for a given element and geometry as a function of the typical dimension (thickness, for foils, and radius, for wires).

Recently, COPLEY[15] studied the scattering effect within an absorbing sphere and, according to his results, the self-shielding factor is determined through a set of curves as a function of the macroscopic absorption and scattering cross sections of the sample. On the other hand, there exist specific formulas to determine G_{th} for various geometries in a two-step process: (1) for a given geometry, calculation of the neutron self-shielding factor considering all the collisions as absorbing collisions, i.e., using the total macroscopic cross section instead of the absorption macroscopic cross section,[16] (2) correction of the neutron scattering effect on the sample by using the Stewart's formula.[17]

The present paper deals with the description of the thermal neutron self-shielding factor for any element/substance/compound and various geometries (foils, wires, spheres and cylinders) by means of a unique universal curve on the basis of a dimensionless variable, which includes the physical, nuclear and geometrical properties of the sample.

Neutron activation

The sample activity, A^*, induced by radiative neutron capture, at the end of the irradiation, can be derived from the Westcott formalism.[18] In many cases, the expression is as follows:

$$A^* = N\left[g\sigma_0\Phi_0 G_{th} + \frac{2}{\sqrt{\pi}}\Phi_{epi}G_{res}(I_\gamma - 0.45\sigma_0)\right]\cdot(1-e^{\lambda t_{irr}}) \quad (1)$$

where N is the number of atoms ^{A}X present in the sample, σ_0 is the thermal neutron cross section for the (n,γ) reaction (v_0 = 2200 m/s), g is a parameter representing the deviation of the (n,γ) cross section in the thermal region from the $1/v$ law, I_γ is the radiative neutron capture resonance integral, $\Phi_0 = n v_0$ is the thermal neutron flux, n being the total neutron density, $\Phi_{epi} = \Phi$ (E = 1 eV) is the epithermal neutron flux, λ is the decay constant of the nuclide ^{A+1}X, t_{irr} is the irradiation time and, as said above, G_{th} is the thermal neutron self-shielding factor, and G_{res} is the resonance neutron self-shielding factor. The aim of this work is related with the calculation of G_{th}.

Calculation

For a given element, geometry and sample dimension, G_{th} is calculated as the ratio between the reaction rates per atom in the real sample and in a similar and infinitely diluted sample:

$$G_{th}(x) = \frac{\int_{E_1}^{E_2} M(E)\,\sigma_a(E)\,dE}{\int_{E_1}^{E_2} M_0(E)\,\sigma_a(E)\,dE} \quad (2)$$

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0236–5731/2004/USD 20.00

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where x is the typical sample dimension, $M_0(E)$ is the non-perturbed thermal (Maxwellian) neutron flux per unit energy interval (inside the infinitely diluted sample), $M(E)$ is the perturbed thermal neutron flux inside the real sample, $\sigma_a(E)$ designates the energy dependent (n,γ) cross section, and E_1 and E_2 are the lower and the upper limits of the thermal neutron spectrum at room temperature, respectively. The total neutron cross section is adopted in the calculation of the perturbed thermal neutron flux, thus taking into account the neutron scattering in the sample. In the calculations, the density for infinite dilution is assumed to be $\rho = 10^{-6}\,\rho_0$, where ρ_0 represents the density of the real sample.

Thermal neutron self-shielding factors for foils, wires, spheres and cylinders of seventeen elements with various properties (Table 1) have been calculated using the MCNP code.[19]

In order to practically eliminate the edge effect of neutrons in the samples, the following conditions have been adopted: (1) $R/t \geq 100$ for foils, where t and R are the circular foil thickness and radius, and (2) $l/R \geq 50$ for wires, where l and R are the wire length and radius, respectively.

Results and discussion

As shown in Fig. 1 for wires, the value of G_{th} depends on the geometrical, physical and nuclear properties of the material as well as on the typical dimension of the sample. Similar results were obtained for foils, spheres and cylinders.

Figure 2 shows G_{th} for cylindrical gold samples as a function of the radius, r, and height, h. For a given radius, G_{th} diminishes as the height increases, as a consequence of a lesser edge effect. Similar results were observed for other elements.

In recent works, the authors have shown that it is possible to introduce a dimensionless variable, z, which converts the dependence of G_{th} on the relevant properties of the samples into a unique curve for foils[22] and for spheres.[23]

The global analysis of the results for the various geometries shows that z is given by:

$$z = x\Sigma_t\left(1 - \frac{\Sigma_s}{\Sigma_t}\right)^k = x\Sigma_t\left(\frac{\Sigma_a}{\Sigma_t}\right)^k \qquad (3)$$

where $k = 0.85 \pm 0.05$, Σ_t, Σ_s and Σ_a are the total, scattering and absorption macroscopic cross sections averaged over the thermal neutron spectrum at room temperature, respectively, and x is the typical dimension of the sample for a given geometry ($x=t$ – foil thickness; $x=R$ – wire or sphere radius; $x=rh/(r+h)$ – r and h: radius and height of the cylinder).[16] Note that the variable z takes into account the neutron scattering in the sample.

Table 1. Physical and nuclear properties of the studied elements[20,21]

Element	A, $g \cdot mol^{-1}$	ρ, $g \cdot cm^{-3}$	σ_t,* b	σ_s,* b	σ_a,* b
Al	26.98	2.7	1.617	1.414	0.203
Au	196.97	19.3	95.05	6.86	88.19
Cd	112.41	8.65	2996.5	11.54	2985.0
Co	58.93	8.9	38.98	6.02	32.96
Cu	63.55	8.96	11.23	7.87	3.36
Eu	151.96	5.24	3655.6	6.61	3649.0
Gd	157.25	7.9	36892.3	138.7	36753.6
In	114.82	7.31	176.8	2.55	174.3
Ir	192.22	22.42	384.2	14.7	369.5
Mo	95.94	10.22	7.77	5.48	2.29
Ni	58.69	8.9	22.58	18.69	3.89
Pb	207.20	11.35	11.22	11.07	0.15
Pt	195.08	21.45	21.5	12.4	9.1
Rh	102.91	12.41	136.2	3.25	132.9
Sc	44.96	2.99	46.54	22.48	24.06
Sm	150.36	7.52	8419.3	52.3	8367.0
Ta	180.95	16.65	24.02	5.64	18.38

* σ_t, σ_s, σ_a are, respectively, the total, scattering and absorption microscopic cross sections averaged over the thermal neutron spectrum at room temperature (293.6 K).

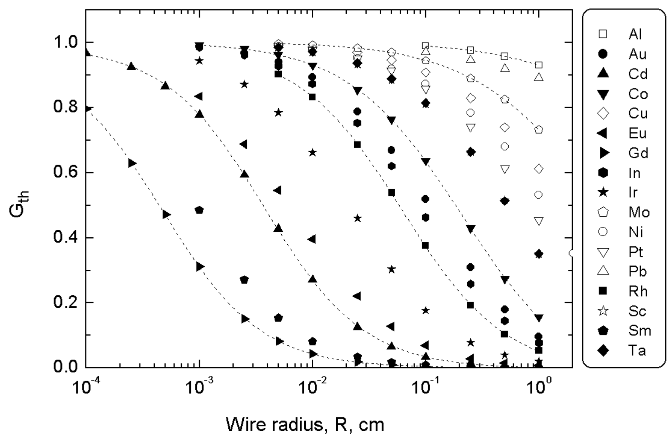

Fig. 1. Thermal neutron self-shielding factor for wires as a function of the wire radius

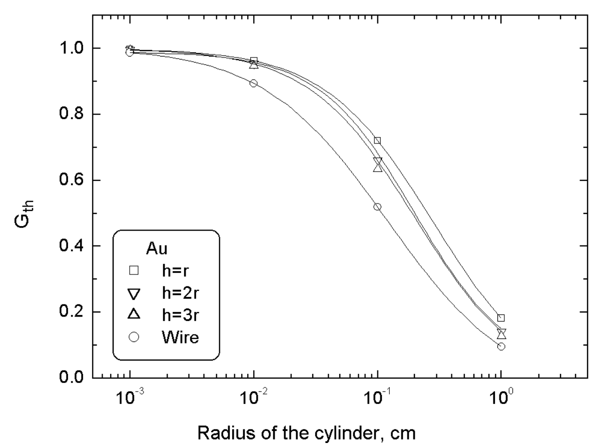

Fig. 2. Thermal neutron self-shielding factor for gold as a function of the cylinder radius, *r*, and height, *h*

The analysis of the results also shows that a sigmoid is the best curve to be fitted to the calculated $G_{th}(z)$ values. The expression of this curve is:

$$G_{th}(z) = \frac{A_1 - A_2}{1 + \left(\frac{z}{z_0}\right)^p} + A_2 \qquad (4)$$

where A_1, A_2, z_0 and p are the curve parameters, to be adjusted to the calculated values.

Figures 3, 4, 5 and 6 show the calculated values of $G_{th}(z)$ of samples in form of foils, wires, spheres and cylinders, respectively, and the curves adjusted to each set of points.

In Table 2 the parameters of the adjusted curves are presented. The analysis of these values shows that $A_1 = 1$ and $A_2 = 0$ due to physical reasons, p is practically constant and z_0 is variable, depending on the sample geometry. We can assume that $z_{0,sph}/z_0$ is equal to 1 for spheres; 1.51 for foils; 2.00 for wires; and 1.62 for cylinders.

Using the variable transformation:

$$y = \frac{z_{0,sph}}{z_0} \cdot x \qquad (5)$$

all the values of G_{th} for foils, wires and and cylinders are translated upon the values of the spheres and a unique curve, the "universal curve", can fit all values, as it is shown in Fig. 7. The parameters of the universal curve are shown in Table 3.

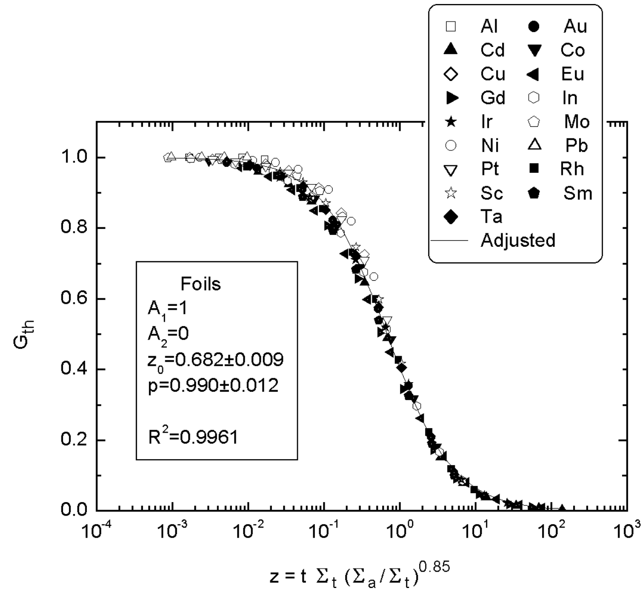

Fig. 3. Thermal neutron self-shielding factor of foil samples as a function of $z = t\,\Sigma_t\,(\Sigma_a/\Sigma_t)^{0.85}$, where t is the foil thickness

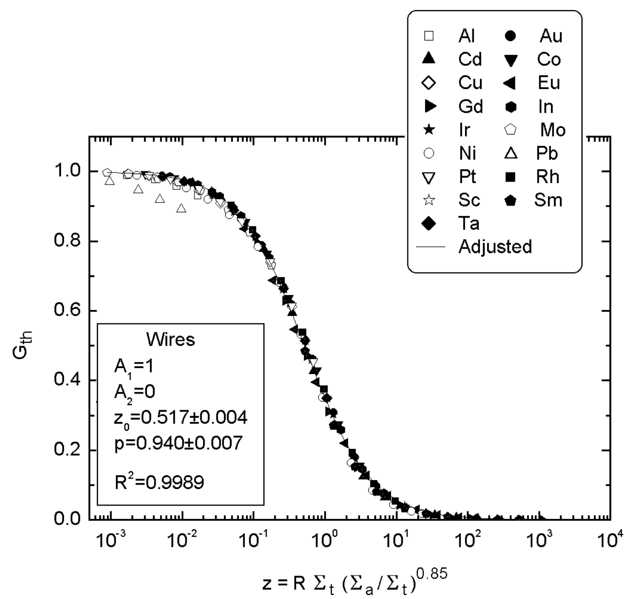

Fig. 4. Thermal neutron self-shielding factor of wire samples as a function of $z = R\,\Sigma_t\,(\Sigma_a/\Sigma_t)^{0.85}$, where R is the wire radius

Then the sigmoid

$$G_{th} = \frac{1}{1+\left(\dfrac{z}{1.029}\right)^{1.009}} \tag{6}$$

can be used to evaluate G_{th} for any new sample (any geometry and composition). Taking into account that z_0 and p are approximatelly equal to 1, G_{th} can be approached by a simpler equation:

$$G_{th} = \frac{1}{1+z} \tag{7}$$

In the range $10^{-4} \le z \le 10$, the differences between the sigmoid and the simpler curve are smaller than 1.5%.

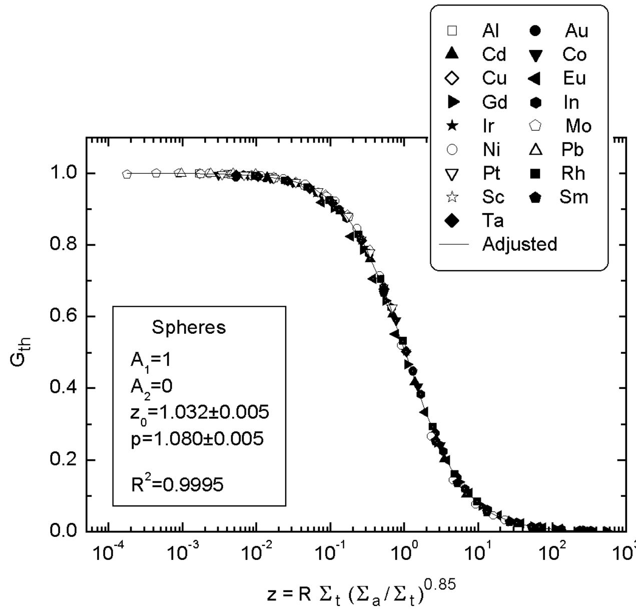

Fig. 5. Thermal neutron self-shielding factor of sphere samples as a function of $z = R\ \Sigma_t\ (\Sigma_a/\Sigma_t)^{0.85}$, where *R* is the sphere radius

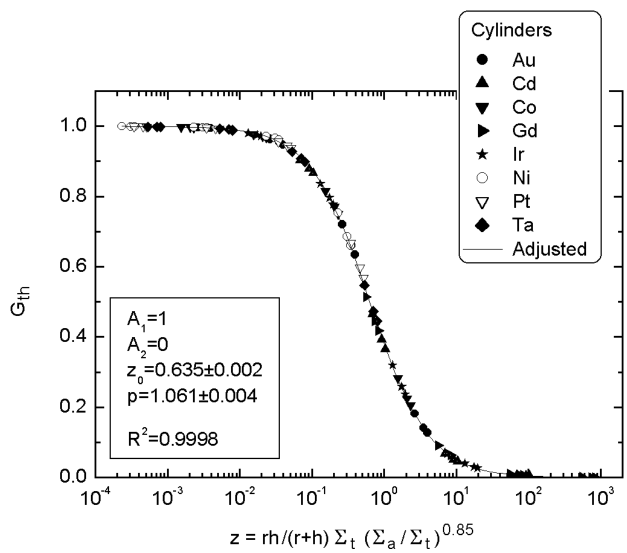

Fig. 6. Thermal neutron self-shielding factor of cylindrical samples as a function of $z = rh/(r+h)\ \Sigma_t\ (\Sigma_a/\Sigma_t)^{0.85}$, where *r* and *h* are the cylinder radius and height, respectively

Table 2. Parameters of the sigmoidal curves

Geometry	Typical dimension *x*, cm	A_1	A_2	z_0	*p*	$z_{0,sph}/z_0$
Foils	*x* = *t*	1	0	0.682	0.990	1.51
Wires	*x* = *R*	1	0	0.517	0.940	2.00
Cylinders	*x* = *rh/(r+h)*	1	0	0.635	1.061	1.62
Spheres	*x* = *R*	1	0	1.032	1.080	1

Table 3. Parameters of the universal curve

Geometry (typical dimensions)	y, cm	A_1	A_2	z_0	p
Foils (thickness = t)	$y = 1.5\,t$				
Wires (radius = R)	$y = 2.0\,R$	1	0	1.029 ± 0.005	1.009 ± 0.005
Cylinders (radius = r; height = h)*	$y = 1.6\,rh/(r+h)$				
Spheres (radius = R)	$y = R$				

* $1 \le h/r \le 3$.

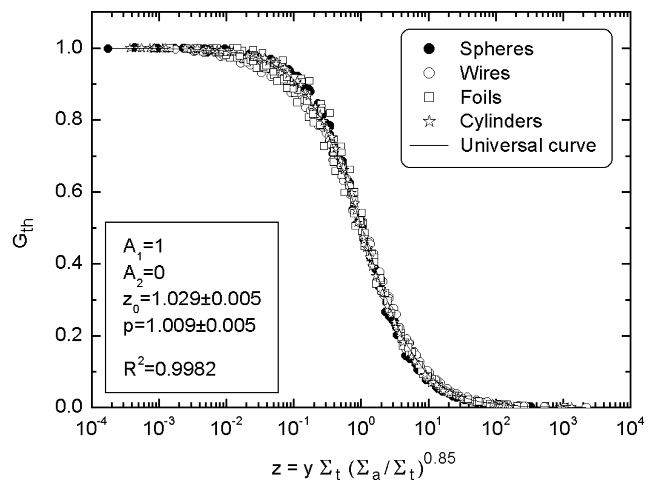

Fig. 7. Universal curve of G_{th} as a function of $z = y\,\Sigma_t\,(\Sigma_a/\Sigma_t)^{0.85}$, where $y = 1.5\,t$ for foils; $y = 2.0\,R$ for wires; $y = 1.6\,rh/(r+h)$ for cylinders; and $y = R$ for spheres

Table 4. Average deviations of the experimental and calculated values relative to the universal curve

Foils, experimental	0.97
Wires, experimental	1.05
Foils, calculated	0.98
Wires, calculated	1.08

Figure 8 shows the comparison of the universal curve with the experimental and calculated values obtained by different authors. Note that the calculated values[14] were obtained for 2200 m/s neutrons, consequently, the corresponding z values were calculated using the 2200 m/s cross sections, instead of the cross sections averaged over the thermal neutron spectrum at room temperature. There is a good agreement. The average deviations of the experimental and calculated values relatively to the universal curve are given in Table 4.

Conclusions

In spite of large differences between the physical and nuclear properties of the studied elements, a universal curve can describe the behavior of the thermal neutron self-shielding factor for foils, wires, spheres and cylinders. In this curve, G_{th} is expressed as a function of a dimensionless variable, which takes into account the physical, nuclear and geometrical properties of the sample. The universal curve is in good agreement with the experimental and calculated values obtained from the literature.

The good agreement between the calculated values[14] and the universal curve means that the self-shielding factor for 2200 m/s neutrons can be estimated by using the corresponding cross sections in the calculation of the variable z, instead of the cross sections averaged over the thermal neutron spectrum at room temperature.

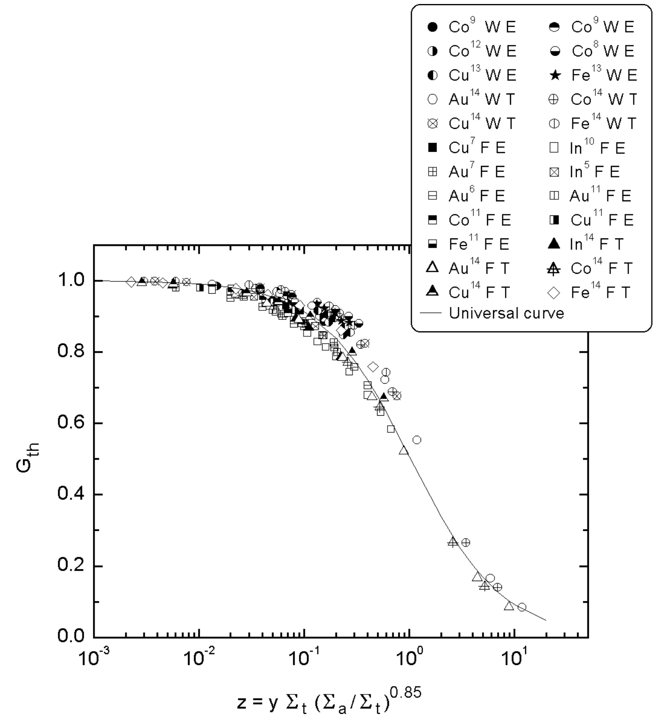

Fig. 8. Comparison of the universal curve of G_{th} with experimental and calculated values from the literature (F – foil; W – wire; E – experimental; T – theoretical)

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