Development of a Unique Curve for Thermal Neutron Self-Shielding Factors in Spherical Scattering Materials

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Abstract – The introduction of an appropriate dimensionless variable, which takes into account, simultaneously, the material absorbing and scattering properties, can express the thermal neutron self-shielding factor of spherical samples by a unique curve. It is shown that the differences between the values calculated by Blaauw and those of the proposed curve are < 1.5%.

1. Introduction

It has been recognized that thermal neutron scattering inside samples has important effects in neutron absorption experiments\(^1\)\(^\text{-}^4\). Its importance may be reflected in the thermal neutron self-shielding factor \(G_{th}\). For a given total cross-section, the scattering of neutron increases the self-shielding factor since a pure scatterer in an isotropic neutron field has no self-shielding\(^2\).

This paper uses the results published by Blaauw\(^2\) and proposes a unique curve that which describes conveniently the behaviour of the thermal neutron self-shielding factor.

2. Blaauw method for calculation of self-shielding factors

Blaauw\(^2\) computes the self-shielding factor \(G_{th}\) of a spherical sample using the Stewart’s formula\(^5\) by a two step calculation. First of all, he computes the self-shielding factor without scattering, \(G_{th}^0\), using the macroscopic total scattering cross-section \(\Sigma_t\) instead of the macroscopic absorption scattering cross-section \(\Sigma_a\) as though the sample did not scatter neutrons. Stewart used only \(\Sigma_a\) to calculate \(G_{th}^0\). Then Blaauw uses the Stewart’s formula to correct for scattering effects. Accordingly, the Blaauw values were calculated using the expressions

\[
G_{th}^0 = \frac{3}{X_t^3} \left[ \frac{X_t^2}{2} - 1 + (1 + X_t) e^{-X_t} \right]
\]

and

\[
G_{th} = \frac{G_{th}^0}{1 - \sum_s \left( 1 - G_{th}^0 \right)} \tag{2}
\]
where

\[ X_i = 2R \Sigma_i \]

\[ R = \text{sphere's radius} \]

\[ \Sigma_i = \text{macroscopic scattering cross-section.} \]

### 3. Results and discussion

Blaauw\(^2\) presents sets of thermal neutron self-shielding factor curves plotted as a function of \(X_a = 2\Sigma_a R\) and \(X_s = 2\Sigma_s R\). \(G_{th}\) have been calculated over a wide range of \(X_a\) and \(X_s\) values between 0 and 10. Figure 1 presents some of these values for \(X_a\) varying between 0 and 10 and \(X_s = 0, 2, 5, 10\). It demonstrates that \(G_{th}\) decreases with \(X_a\) at constant \(X_s\) and with \(X_s\) at constant \(X_a\).

![Fig. 1 – Self-shielding factor calculated by Blaauw, plotted as a function of \(X_a\) for different values of \(X_s\)](image)

In this paper it is proposed to introduce a dimensionless variable, which takes into account, simultaneously, both macroscopic absorption and scattering cross sections. The variable is:

\[ X = 2R \Sigma_i \left( \frac{\Sigma_a}{\Sigma_i} \right)^x = 2R \Sigma_i \left( 1 - \frac{\Sigma_s}{\Sigma_i} \right)^x, \]

\( \text{Eq. (3)} \)
where $x$ is a parameter to be adjusted to the calculated or measured values in order to transform the set of curves into a unique curve. The expression between parenthesis takes into account scattering effects: for a pure scatterer $X = 0$. The results of Blaauw plotted against $X$ are shown in Figures 2, where $x = 0.85$.

![Graph showing Gth as a function of X with data points and a sigmoid curve]

Fig. 2 – Self-shielding factors calculated by Blaauw as a function of $X$, for $x=0.85$, and the adjusted curve ($A_1 = 1; A_2 = 0; z_0 = 2.227 \pm 0.004; p = 1.160 \pm 0.002; r^2 = 0.9999$).

A sigmoid curve:

$$G_{th} = \frac{A_1 - A_2}{1 - \left(\frac{X}{X_0}\right)^p} + A_2$$  \hspace{1cm} (4)$$

was adjusted to the calculated values. The results show that a unique curve can take into account, simultaneously, the dependence of $G_{th}$ on the absorption and scattering cross-sections. The mean and maximum differences between the adjusted curve proposed in this work and the values calculated by Blaauw are 0.2% and 1.5%, respectively.

Figure 3 compares $G_{th}$ calculated by Blaauw with values calculated by Martinho, Salgado, and Gonçalves using the MCNP code for materials with very different absorption and scattering cross sections. A good agreement is obtained between the Blaauw and our values.
4. Conclusions

The results presented in this work show that a unique curve can be adjusted to the calculated (or experimental) values of the thermal neutron self-shielding factor for spherical samples. This is obtained through the introduction of a dimensionless variable that takes into account, simultaneously, the scattering and absorption properties of the samples. The differences between the calculated values and those of the proposed curve are < 1.5%.

References